# Prescription Dive Masks and Power Compensations.

Grant Hannaford BSc SLD (Hons) FBDO BA (Hons)

## Surfaces with F>0 in medium other than air

Immersion of a lens in water presents an interesting case for ray traces as the refractive index of the media surround the front and back surface differs. In this case of dive masks the front surface is refracting in a medium with n=1.333 ( Tunnacliffe, Hirst et al. 1996) and the rear surface is refracting in a medium with n=1.

Working out the lens form for a +2.00 in air (Jalie and Association of Dispensing Opticians (Great Britain) 2016) we may obtain:

$$F'_v = +2.00$$
$$F_1 = +6.00$$
$$t = 2mm = 0.002m$$
$$n = 1.498$$
$$F_2 = F'_v - \frac{F_1}{1-\frac{t}{n}F_1} = +2.00 - \frac{+6.00}{1-\frac{0.002}{1.498}\left(+6.00\right)} = -4.05D$$

Eq (1)

Where F = power in dioptres, $F'_v$ = back vertex power, $F_n$ = power of $n^{th}$ surface, r = radius, t = thickness and n = refractive index.

By calculating the radius of curvature used for $F_1$ in air (2016) and then recalculating the surface power in water we obtain:

$$n = 1.498$$
$$n' = -1.333$$
$$r_{1AIR} = \frac{(1.498 - 1)}{+6.00} = 0.083m$$
$$F_{1WATER} = \frac{(1.498 - 1.333)}{0.083} = +1.99D$$

Eq (2)

This represents a significant reduction in the effective power of the surface indicating that the surface and by extension the lens is no longer effective for the patient. Calculating the back vertex power using the new value for $F_1$ :

$$F'_v = \frac{F_1 + F_2 - \frac{t}{n}F_1F_2}{1 - \frac{t}{n}F_1} = \frac{+1.99 - 4.05 - \frac{0.002}{1.498}(+1.99)(-4.05)}{1 - \frac{0.002}{1.498}(+1.99)} = -2.06D$$

Eq (3)

To maintain the correct back vertex power (ie the one that the patient will require inside the goggle) we must compensate the front vertex power to allow for the reduced power at the front surface.

$$n = 1.333$$
$$n' = 1.498$$
$$F_1 = +6.00$$
$$r_{1WATER=} = \frac{(1.498 - 1.333)}{+6.00} = 0.0275m$$
$$\rightarrow F_{1AIR} = \frac{(1.498 - 1)}{0.0275} = +18.109D$$

Eq (4)

This value for $F_1$ indicates that in air the lens will have a back vertex power of:

$$t = 12mm = 0.12m$$
$$F'_v = \frac{F_1 + F_2 - \frac{t}{n}F_1F_2}{1 - \frac{t}{n}F_1} = \frac{+18.109 - 4.05 - \frac{0.012}{1.498}(+18.109)(-4.05)}{1 - \frac{0.01\ 2}{1.498}(+18.109)} = +17.13D$$

Eq (5)

Resulting in an error of 15.13 dioptres when the lens is worn in air. Clearly this is undesirable for the patient but more importantly it indicates that the moderate ‘rule of thumb’ power correction factors as indicated by industry are not related to correction for the immersing medium.

In order to arrive at an expression for a power correction value we have explored the relationship between the power of the front surface in air and the power of the same surface in water.

Table 1 - Relative surface powers in water and air.

| $F_{1\ AIR}$ (D) | $r_{1Air}$ n=1.498 (m) | $F_{1Water}$ n=1.498 | $\Delta_{F1}$ | $F_{1\ Water}/\Delta_{F1}$ |
|---|---|---|---|---|
| 1 | 0.50 | 0.33 | 0.67 | 0.49 |
| 2 | 0.25 | 0.66 | 1.34 | 0.49 |
| 3 | 0.17 | 0.99 | 2.01 | 0.49 |
| 4 | 0.12 | 1.32 | 2.68 | 0.49 |
| 5 | 0.10 | 1.65 | 3.35 | 0.49 |
| 6 | 0.08 | 1.98 | 4.02 | 0.49 |
| 7 | 0.07 | 2.31 | 4.69 | 0.49 |
| 8 | 0.06 | 2.65 | 5.35 | 0.49 |
| 9 | 0.06 | 2.98 | 6.02 | 0.49 |
| 10 | 0.05 | 3.31 | 6.69 | 0.49 |

Of interest is whether there may be a constant value which may be applied to a surface as a correcting factor to indicate the effective power of the surface in water. Noting that the relationship between the two refractive indices is constant we are readily able to verify this by examining the relationship between the change in power and the new power of the surface in water (Table 1). Similarly, a constant value of 1.49 is found when the difference in power is related to the power of the surface in air. For a material with a refractive index of 1.6 we obtain a value of 0.8 which also indicates that some compensating factor may be achievable for all materials.

An expression for the compensating value is readily obtained as follows. Firstly the expressions for the power of the same surface with respect to each refractive index:

$$F_{1Air} = \frac{\left(n' - n_{Air}\right)}{r_1}$$

$$F_{1Water} = \frac{\left(n' - n_{Water}\right)}{r_1}$$

Eq (6)

Note that the radius of curvature remains unchanged, this indicated the following equality:

$$r_1 = \frac{(n' - n_{Air})}{F_{1Air}}$$

$$r_1 = \frac{(n' - n_{Water})}{F_{1Water}}$$

$$\rightarrow \frac{(n' - n_{Air})}{F_{1Air}} = r_1 = \frac{(n' - n_{Water})}{F_{1Water}}$$

$$\rightarrow \frac{(n' - n_{Air})}{F_{1Air}} = \frac{(n' - n_{Water})}{F_{1Water}}$$

Eq (7)

We are manipulating this to find the expression for the new value of the surface in water so we isolate $F_{1Water}$:

$$\frac{(n' - n_{Air})}{F_{1Air}} = \frac{(n' - n_{Water})}{F_{1Water}}$$

$$\rightarrow \frac{F_{1Water}}{F_{1Air}} = \frac{(n' - n_{Water})}{(n' - n_{Air})}$$

$$\rightarrow F_{1Water} = F_{1Air} \frac{(n' - n_{Water})}{(n' - n_{Air})}$$

Eq (8)

Using values for CR39 (n=1.498) and water (n=1.333):

$$F_{1Water} = F_{1Air} \frac{(n' - n_{Water})}{(n' - n_{Air})} = F_{1Air} \frac{(1.498 - 1.333)}{(1.498 - 1)} = 0.331 F_{1Air}$$

Eq (9)

Table 2 - Comparing to the method used in Eq (2).

| $F_{1\,AIR}$ | $F_{1Water}$ n=1.498 radius calculation | $0.331F_{1Air}$ coefficient method |
|---|---|---|
| 1 | 0.33 | 0.33 |
| 2 | 0.66 | 0.66 |
| 3 | 0.99 | 0.99 |
| 4 | 1.32 | 1.32 |
| 5 | 1.65 | 1.66 |
| 6 | 1.98 | 1.99 |
| 7 | 2.31 | 2.32 |
| 8 | 2.65 | 2.65 |
| 9 | 2.98 | 2.98 |
| 10 | 3.31 | 3.31 |

For a material with refractive index n=1.6 we obtain:

$$F_{1Water} = F_{1Air}\frac{(n'-n_{Water})}{(n'-1)} = F_{1Air}\frac{(1.6-1.333)}{(1.6-1)} = 0.445F_{1Air}$$

Eq (10)

Which again agrees well with the earlier method for calculation.

To calculate the compensated value for the front surface of the lens, ie the power that must be applied to the surface in air to achieve the desired power in water we use the same method as Eq (9) to arrive at:

$$\frac{(n'-n_{Air})}{F_{1Air}} = \frac{(n'-n_{Water})}{F_{1Water}}$$
$$\rightarrow \frac{F_{1Water}}{F_{1Air}} = \frac{(n'-n_{Water})}{(n'-n_{Air})}$$
$$\rightarrow F_{1Air} = F_{1Water}\frac{(n'-n_{Air})}{(n'-n_{Water})}$$

Eq (11)

Which is seen to be the inverse of the coefficient used to calculate the power of the surface when immersed in another medium (water). Results for n=1.6 and n=1.498 are:

$$n' = 1.6$$

$$F_{1Air} = F_{1Water}\frac{(n' - n_{Air})}{(n' - n_{Water})} = F_{1Water}\frac{(1.6 - 1)}{(1.6 - 1.333)} = 2.25F_{1Water}$$

$$n = 1.498$$

$$F_{1Air} = F_{1Water}\frac{(n' - n_{Air})}{(n' - n_{Water})} = F_{1Water}\frac{(1.498 - 1)}{(1.498 - 1.333)} = 3.02F_{1Water}$$

Eq (12)

$F_2$ may then be calculated as Eq (2).

As is seen in this treatment there is limited viability for prescription lenses in dive masks where the interface between the water and the lens contains power. The simplest solution is to retain the plano flat lens that comes with the mask and cement a lens with a plano front surface to the inside of the existing lens. The benefit of this method is that the water no longer has any impact upon the power of the front surface of the lens.

It should be noted that setting the front surface of the lens to plano removes any ability to apply best form to the lens design so these can be prone to more aberrations. However the use of a +18.00 front surface will tend to be an unsatisfactory lens form so the relative impact of the flat front surface is negligible (Table 4)

Table 3 - Comparison of radius and coefficient methods for n=1.6

| $F_{1\ AIR}$ | $F_{1Water}$ n=1.6 | $0.445F_{1Air}$ coefficient method |
|---|---|---|
| 1 | 0.45 | 0.45 |
| 2 | 0.89 | 0.89 |
| 3 | 1.34 | 1.34 |
| 4 | 1.78 | 1.78 |
| 5 | 2.23 | 2.23 |
| 6 | 2.67 | 2.67 |
| 7 | 3.12 | 3.12 |
| 8 | 3.56 | 3.56 |
| 9 | 4.01 | 4.01 |
| 10 | 4.45 | 4.45 |

Table 4 - Effect of two lens forms on lens performance

| | $F_1$=+18.00 | $F_1$=0.00 | Variance |
|---|---|---|---|
| **Spec mag** | 17.1% | 8.7% | -8.4% |
| **Distortion (Dioptres)** | 0.87 | 1.24 | 42.5% |
| **Oblique Astigmatic Error (Dioptres)** | 0.37 | 0.21 | -43.2% |
| **Mean Oblique Error (Dioptres)** | 0.33 | 0.17 | -48.5% |

## Vertex Distance

The majority of ad hoc theories for compensation of power in dive masks utilise the reduction of the power to some extent. These compensations tend to be reductions of 33% (2/3 of original Rx is given) so that a +/-2.00 will be glazed at +/-1.25. This is erroneously attributed to the water environment. In reality any power modification is actually due to the increased back vertex distance and it is generally not of the magnitude indicated in the 2/3 rule. Using the power/distance (Jalie and Association of Dispensing Opticians (Great Britain) 2016) relationship the +2.00 lens mentioned would need to be moved as follows to fit the 2/3 rule (note that d is negative as we are increasing vertex distance):

$$F_E = \frac{F}{1-(-d)F} = \frac{F}{1+dF}$$
$$\rightarrow d = \frac{\left(\frac{F}{F_E}-1\right)}{F} = \frac{\left(\frac{2}{1.25}-1\right)}{2} = 0.3m = 300mm$$

Eq (13)

Which is clearly beyond the reasonable range of back vertex distances for dive masks (30mm-50mm). It is also important to note that reduction of power due to increased vertex distance is only appropriate for plus lenses as minus powers require increased powers for greater vertex distances. A more appropriate compensation for back vertex power is illustrated by the effective powers below:

$$bvd_{mask} = 50mm$$
$$bvd_{test} = 12mm$$
$$d = 50-12 = 38mm = 0.038m$$
$$F = +2.00$$
$$F_E = \frac{F}{1-(-d)F} = \frac{+2.00}{1+0.038(+2.00)} = +1.86D$$
$$F = -2.00$$
$$F_E = \frac{F}{1-(-d)F} = \frac{-2.00}{1+0.038(-2.00)} = -2.16D$$

Eq (14)

As the relationship is non linear the most appropriate method for determining the correct back vertex power is simply to calculate it. There are many charts available to assist in this and the creation of excel documents to assist is a relatively trivial matter.

Table 5 is an example of a table indicating the back vertex power for dive masks with a reference test vertex distance of 12mm used to calculate the change in back vertex distance.

## Conclusion

The entire purpose of dive masks is to place the cornea in air, negating any impact of immersion in media of different indices. Because of this it is important when working with dive masks to identify the arrangement of lenses with relation to the environment. If the front surface of the lens immersed in water is anything other than plano then power compensations will need to be made to the optical system in addition the normal vertex distance compensations. For the most common dive mask designs with a plano front surface the primary compensation will be due to the larger vertex distance, not the immersion in water.

# Bibliography

Atchison, D. A. and G. Smith (2000). Optics of the human eye. Oxford, Butterworth-Heinemann.
Jalie, M. (2008). Ophthalmic lenses and dispensing. Oxford, Butterworth-Heinemann.
Jalie, M. and Association of Dispensing Opticians (Great Britain) (2016). The principles of ophthalmic lenses. London, Association of British Dispensing Opticians.
Tunnacliffe, A. H. (1993). Introduction to visual optics. London, Association of British Dispensing Opticians.
Tunnacliffe, A. H., J. G. Hirst and Association of British Dispensing Opticians. (1996). Optics. Canterbury, Kent, Association of British Dispensing Opticians.

Table 5 - Back vertex power corrections

| | **Plus lens** | | **Power (\|F\|)** | | **Minus lens** | |
|---|---|---|---|---|---|---|
| Vertex distance of mask | | | | Vertex distance of mask | | |
| **50** | **45** | **40** | | **40** | **45** | **50** |
| 0.96 | 0.97 | 0.97 | **1.00** | -1.03 | -1.03 | -1.04 |
| 1.42 | 1.43 | 1.44 | **1.50** | -1.57 | -1.58 | -1.59 |
| 1.86 | 1.88 | 1.89 | **2.00** | -2.12 | -2.14 | -2.16 |
| 2.28 | 2.31 | 2.34 | **2.50** | -2.69 | -2.72 | -2.76 |
| 2.69 | 2.73 | 2.77 | **3.00** | -3.28 | -3.33 | -3.39 |
| 3.09 | 3.14 | 3.19 | **3.50** | -3.88 | -3.96 | -4.04 |
| 3.47 | 3.53 | 3.60 | **4.00** | -4.50 | -4.61 | -4.72 |
| 3.84 | 3.92 | 4.00 | **4.50** | -5.15 | -5.28 | -5.43 |
| 4.20 | 4.29 | 4.39 | **5.00** | -5.81 | -5.99 | -6.17 |
| 4.55 | 4.66 | 4.77 | **5.50** | -6.50 | -6.72 | -6.95 |
| 4.89 | 5.01 | 5.14 | **6.00** | -7.21 | -7.48 | -7.77 |
| 5.21 | 5.35 | 5.50 | **6.50** | -7.95 | -8.27 | -8.63 |
| 5.53 | 5.69 | 5.85 | **7.00** | -8.71 | -9.10 | -9.54 |
| 5.84 | 6.01 | 6.20 | **7.50** | -9.49 | -9.97 | -10.49 |
| 6.13 | 6.33 | 6.54 | **8.00** | -10.31 | -10.87 | -11.49 |
| 6.42 | 6.64 | 6.87 | **8.50** | -11.15 | -11.81 | -12.56 |
| 6.71 | 6.94 | 7.19 | **9.00** | -12.03 | -12.80 | -13.68 |
| 6.98 | 7.23 | 7.50 | **9.50** | -12.94 | -13.84 | -14.87 |
| 7.25 | 7.52 | 7.81 | **10.00** | -13.89 | -14.93 | -16.13 |
| 7.51 | 7.80 | 8.11 | **10.50** | -14.87 | -16.07 | -17.47 |
| 7.76 | 8.07 | 8.41 | **11.00** | -15.90 | -17.27 | -18.90 |
| 8.00 | 8.34 | 8.70 | **11.50** | -16.96 | -18.53 | -20.43 |
| 8.24 | 8.60 | 8.98 | **12.00** | -18.07 | -19.87 | -22.06 |
| 8.47 | 8.85 | 9.26 | **12.50** | -19.23 | -21.28 | -23.81 |
| 8.70 | 9.10 | 9.53 | **13.00** | -20.44 | -22.77 | -25.69 |
| 8.92 | 9.34 | 9.80 | **13.50** | -21.70 | -24.35 | -27.72 |
| 9.14 | 9.58 | 10.06 | **14.00** | -23.03 | -26.02 | -29.91 |
| 9.35 | 9.81 | 10.31 | **14.50** | -24.41 | -27.80 | -32.29 |
| 9.55 | 10.03 | 10.56 | **15.00** | -25.86 | -29.70 | -34.88 |